

Ground State Energy of Massive Scalar Field Inside a Spherical Region in the Global Monopole Background

E.R. Bezerra de Mello ^{*}, V.B. Bezerra [†], and N.R. Khusnutdinov [‡]

Departamento de Física, Universidade Federal da Paraíba,

Caixa Postal 5008, CEP 58051-970 João Pessoa, Pb, Brazil

Abstract

Using the zeta function regularization method we calculate the ground state energy of scalar massive field inside a spherical region in the space-time of a point-like global monopole. Two cases are investigated: (i) We calculate the Casimir energy inside a sphere of radius R and make an analytical analysis about it. We observe that this energy may be positive or negative depending on metric coefficient α and non-conformal coupling ξ . In the limit $R \rightarrow \infty$ we found a zero result. (ii) In the second model we surround the monopole by additional sphere of radius $r_0 < R$ and consider scalar field confined in the region between these two spheres. In the latter, the ground state energy presents an additional contribution due to boundary at r_0 which is divergent for small radius. Additional comments about renormalization are considered.

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^{*}e-mail: emello@fisica.ufpb.br

[†]e-mail: valdir@fisica.ufpb.br

[‡]On leave from Kazan State Pedagogical University, Kazan, Russia; e-mail: nail@ntp.ksu.ras.ru

I. INTRODUCTION

Different types of topological objects may have been formed during Universe expansion, these include domain walls, cosmic strings and monopoles [1]. These topological defects appear as a consequence of breakdown of local or global gauge symmetries of a system composed by self-coupling iso-scalar Higgs fields Φ^a . Global monopoles are created due to phase transition when a global gauge symmetry is spontaneously broken and they may have been important for cosmology and astrophysics. The process of global monopole creation is accompanied by particles production [2]. Grand Unified Theory predicts great number of these objects in the Universe [3] but this problem may be avoided using inflationary models. From astrophysical point of view there is at most one global monopole in the local group of galaxies [4].

The space-time of a global monopole in a $O(3)$ broken symmetry model has been investigated by Barriola and Vilenkin [5]. They have shown that far from the compact monopole's core the space-time is approximately described by spherical symmetric metric with an additional solid angle deficit. It is also possible to find solution for the Einstein equation coupled with an energy-momentum tensor associated with a pointlike global monopole. For simplicity we shall consider in this paper this singular configuration. In Ref. [6] a simplified model is presented in order to consider some internal structure for the global monopole.

The analysis of quantum fields on the global monopole background have been considered in Refs. [7] - [9]. It was shown, taking into account only dimensional and conformal considerations [7], that the vacuum expectation value of the energy-momentum tensor associated with a collection of conformal massless quantum field of arbitrary spin in this background has the following general structure

$$\langle T_k^i \rangle = S_k^i \frac{\hbar c}{r^4} ,$$

where the quantities S_k^i depend only on the solid angle deficit and spin of the fields. For scalar field this tensor was investigated more carefully by Mazzitelli and Lousto [8] and for massless spinor field by authors [9] in great details.

The above energy-momentum tensor has non-integrable singularity at origin therefore the ground state energy cannot be found by integrating the energy density. The same problem also appears for cosmic string space-time [10] and in Minkowsky one with boundary condition on dihedral angle [11]. The calculation of ground state energy for the cosmic string space-time was considered in Refs. [12,13] using different approaches. For infinitely thin cosmic string, specific global effect appears which leads to additional surface renormalization [12]. The ground state energy of massive scalar field in the background of a cosmic string with internal nonsingular structure has been considered in Ref. [13]. It has been found that it is zero for arbitrary transverse diameter of the string.

The nontrivial topological structure of space-time leads to a number of interesting effects which are not presented in a flat space. For example, there appear self-interacting forces on a massive point-like particles at rest. These forces have been investigated in Refs. [14,15] for cosmic string and global monopole space-times respectively.

In the framework of zeta function regularization method [16] (see also [17]) the ground state energy of scalar massive field can be obtained by

$$E(s) = \frac{1}{2} M^{2s} \zeta_A(s - \frac{1}{2}) , \quad (1)$$

which is expressed in terms of the zeta function ζ_A associated with the Laplace operator $\hat{A} = -\Delta + \xi \mathcal{R} + m^2$ defined in the three dimensional spatial section of the space-time. Here, the parameter M , with dimension of mass, has been introduced in order to give the correct dimension for the energy. In order to calculate the renormalized ground state energy we shall use the approach which was suggested and developed in Refs. [18–21].

In this paper we would like to discuss the ground state energy of scalar massive field in the background of point-like global monopole space-time inside a spherical region considering an arbitrary non-minimal coupling of this field with the geometry. Because the energy-momentum tensor has non-integrable singularity at the origin we would like to investigate two cases: (i) In the first we consider a point-like global monopole and calculate ground state energy using zeta function approach; (ii) in the second one, we consider a sphere surrounding

the monopole and cut out internal part of it by an appropriate boundary condition for radial functions. This procedure permits us to reveal the role of the singularity. In the limit of zero radius of inner sphere, this model corresponds to topological defects because there is no internal structure of monopole for arbitrary radius of sphere.

The zeta function of Laplace operator on the point-like global monopole background has been considered by Bordag, Kirsten and Dowker in Ref. [22] using the method given in Refs. [18–21]. There, the general mathematical structure of zeta function and the heat kernel coefficients on the generalized cone have been obtained. Because the main emphasis of the present paper is on the ground state energy, we shall rederive in Sec.III some specific formulas for our case which was not considered in Ref. [22].

The organization of this paper is as follows. In Sec.II we briefly review some geometrical properties about global monopole space-time which will be needed. In Sec.III, the zeta function of the Laplace operator on three dimensional section of a point-like global monopole space-time is developed. In Sec.IV we consider the zeta function for global monopole space-time cutting out by the sphere around the origin. In Sec.V, the ground state energy of massive scalar field with arbitrary non-conformal coupling on global monopole background is considered for both above cases. In Sec.VI, we discuss our results. The signature of the space-time, the sign of Riemann and Ricci tensors are the same as in Christensen paper [23]. We use units $\hbar = c = G = 1$.

II. THE GEOMETRY

Global monopoles are heavy objects probably formed in the early Universe by the phase transition which occur in a system composed by a scalar self - coupling triplet field ϕ^a whose original global symmetry $O(3)$ is spontaneously broken to $U(1)$.

The simplest model which gives rise a global monopole is described by the Lagrangian density below

$$L = \frac{1}{2}(\partial_l \phi^a)(\partial^l \phi^a) - \frac{\lambda}{4}(\phi^a \phi^a - \eta^2)^2 .$$

Coupling this matter field with the Einstein equation, Barriola and Vilenkin [5] have shown that the effect produced by this object in the geometry can be approximately represented by a solid angle deficit in the $(3 + 1)$ - dimensional space-time, whose line element is given by

$$ds^2 = -dt^2 + \alpha^{-2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) , \quad (2)$$

where the parameter $\alpha^2 = 1 - 8\pi\eta^2$ is smaller than unity and depend on the symmetry breaking energy scale η . The solid angle in the geommetry defined by (2) is $4\pi\alpha^2$, consequently smaller than 4π . So this spacetime presents a solid angle deficit given by $\delta\Omega = 32\pi^2\eta^2$. We also can note that it is not flat. The nonzero components of Riemann and Ricci tensors, and scalar curvature are given below

$$\mathcal{R}_{\cdot\cdot\theta\varphi}^{\theta\varphi} = \mathcal{R}_{\theta}^{\theta} = \mathcal{R}_{\varphi}^{\varphi} = \frac{1 - \alpha^2}{r^2} , \quad \mathcal{R} = \frac{2(1 - \alpha^2)}{r^2} .$$

For further application let us consider extrinsic curvature tensor on the sphere of radius R around the origin

$$K_{ij} = \nabla_i N_j .$$

Here N_j is outward unit normal vector with coordinates $N_j = (0, \alpha, 0, 0)$. This tensor has two nonzero components

$$K_{\theta}^{\theta} = K_{\varphi}^{\varphi} = \frac{\alpha}{R} .$$

III. ZETA FUNCTION FOR POINT-LIKE GLOBAL MONOPOLE SPACE-TIME

In order to calculate the ground state energy given by Eq.(1) we have to obtain the zeta function of the operator \hat{A} in the neighborhood of point $s = -1/2$. For the calculation of zeta function we follow Refs [19,20,22]. The zeta function of the operator $\hat{A} = -\Delta + \xi\mathcal{R} + m^2$ is defined in terms of the sum over all eigenvalues of this operator by

$$\zeta_A(s - \frac{1}{2}) = \sum_{(n)} (\lambda_{(n)}^2 + m^2)^{1/2-s} .$$

Here $\lambda_{(n)}^2$ is the eigenvalue of operator $\hat{B} = \hat{A} - m^2$. The eigenfunctions of the operator \hat{A} defined in (2) which are regular at the origin have the form

$$\Phi(\mathbf{r}) = \sqrt{\frac{\lambda}{\alpha r}} Y_{lm}(\theta, \varphi) J_\mu\left(\frac{\lambda}{\alpha} r\right) , \quad (3)$$

where Y_{lm} are the spherical harmonics and J_μ is the Bessel function of the first kind with index

$$\mu = \frac{1}{\alpha} \sqrt{(l + \frac{1}{2})^2 + 2(1 - \alpha^2)(\xi - \frac{1}{8})} . \quad (4)$$

A discret set of eigenvalues $\lambda_{l,j}$ can be found applying some boundary condition imposed on this function. Let us consider the Dirichlet boundary condition at the surface of a sphere of radius R concentric with the pointlike monopole

$$\sqrt{\lambda_{l,j}} J_\mu\left(\frac{\lambda_{l,j}}{\alpha} R\right) = 0 . \quad (5)$$

Then, the zeta function reads

$$\zeta_A^R(s - \frac{1}{2}) = \sum_{l=0}^{\infty} \sum_{j=0}^{\infty} (2l+1) (\lambda_{l,j}^2 + m^2)^{1/2-s} ,$$

where the label R in the zeta function was introduced to indicate this kind of boundary condition. The solutions $\lambda_{l,j}$ of equation (5) can not be found in closed form. For this reason we use the method suggested in Refs. [18–20] which allows us to express the zeta function in terms of the eigenfunctions. According to this approach, the sum over j may be converted into contour integral in complex λ -plane using the principal of argument, namely

$$\zeta_A^R(s - \frac{1}{2}) = \sum_{l=0}^{\infty} (2l+1) \int_{\gamma} d\lambda^2 (\lambda^2 + m^2)^{1/2-s} \frac{\partial}{\partial \lambda} \ln \lambda^{-\mu} J_\mu\left(\frac{\lambda}{\alpha} R\right) ,$$

where the contour γ runs counterclockwise and must enclose all solutions of Eq.(5) on positive real axis. Shifting the contour to the imaginary axis we obtain the following formula for the zeta function (see [19] for details)

$$\zeta_A^R(s - \frac{1}{2}) = -\frac{\cos \pi s}{\pi} \sum_{l=0}^{\infty} (2l+1) \int_m^{\infty} dk (k^2 - m^2)^{1/2-s} \frac{\partial}{\partial k} \ln k^{-\mu} I_{\mu}(\frac{k}{\alpha} R) . \quad (6)$$

Here I_{μ} is the modified Bessel function. Let us use the uniform expansion for the Bessel function $I_{\mu}(\mu z)$ as below

$$I_{\mu}(\mu z) = \sqrt{\frac{t}{2\pi\mu}} e^{\mu\eta(z)} \left\{ 1 + \sum_{k=1}^{\infty} \frac{u_k(t)}{\mu^k} \right\} , \quad (7)$$

where $t = 1/\sqrt{1+z^2}$, $\eta(z) = \sqrt{1+z^2} + \ln(z/(1+\sqrt{1+z^2}))$ and $z = kR/\mu\alpha$. The firsts coefficients $u_k(t)$ and the recursion relations for higher ones are listed in [24]. This uniform expansion leads to power series over m , and the term u_N gives the contribution $\sim 1/m^{3-N}$.

We shall make the calculations up to $N = 3$. In this case we obtain the following formula for uniform expansion of the logarithm of Bessel function

$$\ln(k^{-\mu} I_{\mu}(\frac{k}{\alpha} R)) = \mu(\eta(z) - z) - \frac{1}{4} \ln(1+z^2) + \frac{1}{\mu} D_1(t) + \frac{1}{\mu^2} D_2(t) + \frac{1}{\mu^3} D_3(t) , \quad (8)$$

where

$$\begin{aligned} D_1(t) &= \sum_{a=0}^1 x_{1,a} t^{1+2a} = \frac{1}{8} t - \frac{5}{24} t^3 , \\ D_2(t) &= \sum_{a=0}^2 x_{2,a} t^{2+2a} = \frac{1}{16} t^2 - \frac{3}{8} t^4 + \frac{5}{16} t^6 , \\ D_3(t) &= \sum_{a=0}^3 x_{3,a} t^{3+2a} = \frac{25}{384} t^3 - \frac{531}{640} t^5 + \frac{221}{128} t^7 - \frac{1105}{1152} t^9 . \end{aligned} \quad (9)$$

In the above expression we omit all constants which are not important for the calculation of zeta function. Adding and subtracting uniform expansion (8) in integrand of formula (6) we may represent the zeta function in the form

$$\zeta_A^R(s - \frac{1}{2}) = N^R(s) + \frac{m^{-2s}}{(4\pi)^{3/2} \Gamma(s - \frac{1}{2})} \sum_{k=-1}^3 A_k(s, R) , \quad (10)$$

where

$$\begin{aligned} N^R(s) &= -\frac{\cos \pi s}{\pi R} \sum_{l=0}^{\infty} (2l+1) \mu \alpha \int_{\beta/\mu\alpha}^{\infty} dx \left\{ x^2 - \left(\frac{\beta}{\mu\alpha} \right)^2 \right\}^{1/2-s} \\ &\times \frac{\partial}{\partial x} \left\{ \ln I_{\mu}(\mu x) - \mu\eta(x) + \frac{1}{4} \ln(1+x^2) - \frac{1}{\mu} D_1(t) - \frac{1}{\mu^2} D_2(t) - \frac{1}{\mu^3} D_3(t) \right\} , \end{aligned} \quad (11)$$

$$A_{-1}(s, R) = \frac{4\pi^{3/2}m\beta}{\alpha} \sum_{l=0}^{\infty} (2l+1) \left[\frac{\Gamma(s-1)}{\sqrt{\pi}} {}_2F_1 - \frac{\alpha\mu}{\beta} \Gamma(s - \frac{1}{2}) \right] , \quad (12)$$

$$A_0(s, R) = -2\pi^{3/2}m\mathcal{Z}(0, s - \frac{1}{2}) , \quad (13)$$

$$A_1(s, R) = -\frac{\pi m\alpha}{\beta} \left[\mathcal{Z}(0, s) - \frac{10}{3} \mathcal{Z}(2, s+1) \right] , \quad (14)$$

$$A_2(s, R) = -\frac{\pi^{3/2}m\alpha^2}{2\beta^2} \left[\mathcal{Z}(0, s + \frac{1}{2}) - 6\mathcal{Z}(2, s + \frac{3}{2}) + \frac{5}{2} \mathcal{Z}(4, s + \frac{5}{2}) \right] , \quad (15)$$

$$A_3(s, R) = -\frac{\pi m\alpha^3}{24\beta^3} \left[25\mathcal{Z}(0, s+1) - \frac{1062}{5} \mathcal{Z}(2, s+2) + \frac{884}{5} \mathcal{Z}(4, s+3) - \frac{1768}{63} \mathcal{Z}(6, s+4) \right] . \quad (16)$$

Here ${}_2F_1 = {}_2F_1(-\frac{1}{2}, s-1; \frac{1}{2}; -(\frac{\mu\alpha}{\beta}))^2$ is the hypergeometric function, $\beta = mR$ and

$$\mathcal{Z}(p, s) = \Gamma(q) \sum_{l=0}^{\infty} \frac{2l+1}{(1 + \alpha^2\mu^2/\beta^2)^s} \left(\frac{\alpha\mu}{\beta} \right)^p . \quad (17)$$

The series in Eq.(12)

$$T(s) = \sum_{l=0}^{\infty} (2l+1) \left[\frac{\Gamma(s-1)}{\sqrt{\pi}} {}_2F_1 - \frac{\alpha\mu}{\beta} \Gamma(s - \frac{1}{2}) \right] , \quad (18)$$

can be expressed in terms of the same function given in Eq. (17). Indeed, one can use analytical continuation of the hypergeometrical function [24]

$$\begin{aligned} {}_2F_1 \left(-\frac{1}{2}, s-1; \frac{1}{2}; -\left(\frac{\alpha\mu}{\beta} \right)^2 \right) &= \frac{\alpha\mu}{\beta} \frac{\Gamma(1/2)\Gamma(s-1/2)}{\Gamma(s-1)} \\ &+ \frac{\Gamma(1/2)\Gamma(1/2-s)}{\Gamma(-1/2)\Gamma(3/2-s)} \left(1 + \left(\frac{\alpha\mu}{\beta} \right)^2 \right)^{1-s} {}_2F_1 \left(1, s-1; s + \frac{1}{2}; \frac{1}{1 + \left(\frac{\alpha\mu}{\beta} \right)^2} \right) . \end{aligned}$$

So, the first term in the rhs. of the above equation cancels the second one, divergent term in the sum (18) which is due to term $k^{-\mu}$ in (6) (see Ref. [13]). Now, one can use power series expansion for the hypergeometric function because its argument $1/(1 + (\alpha\mu/\beta)^2)$ is always smaller than unity, so we get

$$T(s) = \frac{1}{2\sqrt{\pi}} \Gamma(s-1/2) \sum_{l=0}^{\infty} \frac{\mathcal{Z}(0, n+s-1)}{\Gamma(n+s+1/2)} . \quad (19)$$

Therefore in order to calculate of the zeta function we have to obtain an analytical continuation to the series $\mathcal{Z}(p, q)$. In fact we may consider only

$$\mathcal{Z}(0, s) = \Gamma(q) \sum_{l=0}^{\infty} \frac{2l+1}{(1 + \alpha^2 \mu^2 / \beta^2)^s} , \quad (20)$$

because the other functions with $p = 2, 4, 6, \dots$ can be expressed in terms of $\mathcal{Z}(0, q)$ only. Substituting the value for μ given in Eq.(4) into Eq.(20) we obtain

$$\mathcal{Z}(0, s) = 2\Gamma(q)\beta^{2s} \sum_{l=0}^{\infty} \frac{l+1/2}{((l+1/2)^2 + b^2)^s} , \quad (21)$$

where $b^2 = \beta^2 + 2(1 - \alpha^2)(\xi - 1/8)$. This series is convergent for $\Re q > 1$. It is no difficult to obtain the analytical continuation of this series for small value of parameter b . Indeed, expanding \mathcal{Z} in powers of b we have

$$\mathcal{Z}(0, s) = 2\beta^{2s} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \Gamma(k+s) b^{2k} \zeta_H(2k+2s-1, \frac{1}{2}) . \quad (22)$$

For analytical continuation of this function in the domain $\Re q \leq 1$ and for great value of b , let us consider the series below

$$F(s, a, b^2) = \sum_{l=0}^{\infty} \frac{1}{((l+a)^2 + b^2)^s} .$$

This series, which has been considered in great detail by Elizalde [25], presents the following analytical continuation for great b

$$\begin{aligned} F(s, a, b^2) &\simeq \frac{b^{-2s}}{\Gamma(s)} \sum_{l=0}^{\infty} \frac{(-1)^l \Gamma(l+s)}{l!} b^{-2l} \zeta_H(-2l, a) + \frac{\sqrt{\pi} \Gamma(s-1/2)}{2\Gamma(s)} b^{1-2s} \\ &\quad - \frac{2\pi b^{-1/2-s}}{\Gamma(s)} \sum_{n=1}^{\infty} n^{s-1/2} \cos(2\pi na) K_{s-1/2}(2\pi nb) . \end{aligned}$$

Here ζ_H is the Hurwitz zeta function and K_n is the modified Bessel function. Differentiating this series with respect to a and putting $a = 1/2$ we obtain the analytical continuation that we need, which is the following

$$\sum_{l=0}^{\infty} \frac{l+1/2}{((l+1/2)^2 + b^2)^s} \simeq \frac{b^{2-2s}}{2(s-1)} + \sum_{l=0}^{\infty} \frac{(-1)^l \Gamma(l+s)}{l! \Gamma(s)} b^{-2s-2l} \zeta_H(-1-2l, 1/2) .$$

Taking into account this expression we obtain analytical continuation for function $\mathcal{Z}(0, q)$:

$$\mathcal{Z}(0, s) \simeq \left(\frac{b^2}{\beta^2} \right)^{-s} \left\{ b^2 \Gamma(s-1) + 2 \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \Gamma(l+s) b^{-2l} \zeta_H(-1-2l, 1/2) \right\} , \quad (23)$$

where $b^2/\beta^2 = 1 + 2(1 - \alpha^2)(\xi - 1/8)/\beta^2$. This function has simple poles for integer numbers $q = 1, 0, -1, -2, \dots$. In order to obtain a renormalized value for the ground state energy we have to extract from our expression for zeta function (10) the part which survives in the limit $m \rightarrow \infty$. Moreover to calculate the zeta function up to degree m^0 we need only two terms from series (23) in which $\zeta_H(-1, 1/2) = 1/24$, $\zeta_H(-3, 1/2) = -7/960$ and three terms of $T(s)$ which are given by Eq.(19).

Putting this expression into Eq.(19), and Eqs.(12) - (16) and expanding over $1/\beta = 1/mR \ll 1$ and s , also collecting terms with similar degree on the mass m up to m^0 (we cannot here collect higher orders of m because we used uniform expansion up to this power) we get

$$\begin{aligned} \zeta_A^R(s - \frac{1}{2}) = \frac{m^{-2s}}{(4\pi)^{3/2}} & \left\{ \left[\frac{4\pi R^3}{3\alpha} \right] m^4 \frac{\Gamma(s-2)}{\Gamma(s-\frac{1}{2})} + [-2\pi^{3/2}R^2] m^3 \frac{\Gamma(s-\frac{3}{2})}{\Gamma(s-\frac{1}{2})} \right. \\ & + \left[\frac{7}{3}\pi\alpha R - \frac{4\pi R}{\alpha}(\Delta - \frac{1}{12}) \right] m^2 \frac{\Gamma(s-1)}{\Gamma(s-\frac{1}{2})} + \left[2\pi^{3/2}(\Delta - \frac{1}{12}) \right] m \\ & \left. + \left[\frac{\pi\alpha}{R}(\Delta - \frac{1}{12} + \frac{229}{2520}\alpha^2) - \frac{2\pi}{\alpha R}(\Delta^2 - \frac{1}{6}\Delta + \frac{7}{240}) \right] \frac{\Gamma(s)}{\Gamma(s-\frac{1}{2})} + \dots \right\}. \end{aligned} \quad (24)$$

Here $\Delta = 2(1 - \alpha^2)(\xi - 1/8)$. All these terms are poles contributions in zeta function, all next terms will be finite for $s \rightarrow 0$. Comparing the above expression with that obtained by the Mellin transformation over trace of heat kernel (in three dimensions)

$$\begin{aligned} \zeta_A^R(s - 1/2) = \frac{1}{\Gamma(s - \frac{1}{2})} \int_0^\infty dt t^{s-3/2} K(t) = \frac{m^{-2s}}{(4\pi)^{3/2}} & \left\{ B_0^R m^4 \frac{\Gamma(s-2)}{\Gamma(s-\frac{1}{2})} \right. \\ & + B_{\frac{1}{2}}^R m^3 \frac{\Gamma(s-\frac{3}{2})}{\Gamma(s-\frac{1}{2})} + B_1^R m^2 \frac{\Gamma(s-1)}{\Gamma(s-\frac{1}{2})} + B_{\frac{3}{2}}^R m + B_2^R \frac{\Gamma(s)}{\Gamma(s-\frac{1}{2})} + \dots \left. \right\}, \end{aligned} \quad (25)$$

we obtain the heat kernel coefficients :

$$\begin{aligned} B_0^R &= \frac{4\pi R^3}{3\alpha}, \quad B_{\frac{1}{2}}^R = -2\pi^{3/2}R^2, \quad B_1^R = \frac{7}{3}\pi\alpha R - \frac{4\pi R}{\alpha}(\Delta - \frac{1}{12}), \\ B_{\frac{3}{2}}^R &= 2\pi^{3/2}(\Delta - \frac{1}{12}), \quad B_2^R = \frac{\pi\alpha}{R}(\Delta - \frac{1}{12} + \frac{229}{2520}\alpha^2) - \frac{2\pi}{\alpha R}(\Delta^2 - \frac{1}{6}\Delta + \frac{7}{240}). \end{aligned} \quad (26)$$

Those terms which are proportional to inverse degree of α come from exponential part of the uniform expansion given by (7), and respectively for $T(s)$ (19). The terms which are linear in α^1 or α^0 come from the series $\sum u_k/\mu^k$ in (7).

Now we may compare our results with well-known formulas given in Refs. [26,22,17,27]. The coefficients B_0^R , $B_{\frac{1}{2}}^R$, B_1^R , $B_{\frac{3}{2}}^R$ coincide with general formulas in three dimensions (all geometrical quantities are given in Sec.II)

$$\begin{aligned}
B_0^R &= \frac{4\pi R^3}{3\alpha} = \int_V dV , \\
B_{\frac{1}{2}}^R &= -2\pi^{3/2} R^2 = -\frac{\sqrt{\pi}}{2} \int_{\partial V} dS , \\
B_1^R &= \frac{7}{3}\pi\alpha R - \frac{4\pi R}{\alpha} \left(\Delta - \frac{1}{12}\right) = \left(\frac{1}{6} - \xi\right) \int_V \mathcal{R} dV + \frac{1}{3} \int_{\partial V} (tr K) dS , \\
B_{\frac{3}{2}}^R &= 2\pi^{3/2} \left(\Delta - \frac{1}{12}\right) \\
&= -\frac{\sqrt{\pi}}{192} \int_{\partial V} \left(-96\xi\mathcal{R} + 16\mathcal{R} + 8\mathcal{R}_{ik}N^iN^k + 7(trK)^2 - 10(trK)^2\right) dS .
\end{aligned} \tag{27}$$

However some problems are connected with the term B_2 . The general structure of this term is the following (see [23,27], for example)

$$B_2^R = \int_V b_2 dV + \int_{\partial V} c_2 dS , \tag{28}$$

where

$$b_2 = -\frac{1}{180}\mathcal{R}^{ik}\mathcal{R}_{ik} + \frac{1}{180}\mathcal{R}^{iklj}\mathcal{R}_{iklj} + \frac{1}{6}\left(\frac{1}{5} - \xi\right)\square\mathcal{R} + \frac{1}{2}\left(\frac{1}{6} - \xi\right)^2\mathcal{R}^2 , \tag{29}$$

is volume part, and

$$\begin{aligned}
c_2 &= \frac{1}{3}\left(\frac{1}{6} - \xi\right)\mathcal{R}(trK) + \frac{1}{3}\left(\frac{3}{20} - \xi\right)\mathcal{R}_{;l}N^l - \frac{1}{90}\mathcal{R}_{lk}N^lN^k(trK) + \frac{1}{30}\mathcal{R}_{iljk}N^lN^kK^{ij} \\
&\quad - \frac{1}{90}\mathcal{R}_{il}K^{il} + \frac{1}{315}\left[\frac{5}{3}(trK)^3 - 11(trK)(trK^2) + \frac{40}{3}(trK^3)\right] + \frac{1}{15}\square(trK) ,
\end{aligned} \tag{30}$$

is boundary contribution. Taking into account the results obtained in Sec.II we have these terms in manifest form:

$$b_2 = -\frac{1}{r^4}\frac{\alpha}{4\pi}\left\{\pi\alpha\left(\Delta - \frac{1}{12} + \frac{17}{120}\alpha^2\right) - \frac{2\pi}{\alpha}\left(\Delta^2 - \frac{1}{6}\Delta + \frac{7}{240}\right)\right\} , \tag{31}$$

$$c_2 = -\frac{4\alpha^3}{315R^3} . \tag{32}$$

We observe that the b_2 is proportional to $1/r^4$ and the integral over volume in Eq. (28) will diverge at origin. This problem has already been discussed by Cheeger [28], Brüning

and Seeley [29] and Bordag, Kirsten and Dowker [22] using *partie finite* of the integral. We regularize the expression for B_2 by restricting domain of radial integration

$$B_2 = - \int_{\varepsilon}^R \frac{dr}{r^2} \left\{ \pi\alpha \left(\Delta - \frac{1}{12} + \frac{17}{120}\alpha^2 \right) - \frac{2\pi}{\alpha} \left(\Delta^2 - \frac{1}{6}\Delta + \frac{7}{240} \right) \right\} - \frac{16\pi\alpha^3}{315R}. \quad (33)$$

After integration we take its finite remainder parts as $\varepsilon \rightarrow 0$, and the expression obtained in this way coincides with that given in Eq.(26).

Our expressions for the heat kernel coefficients also agree with that ones obtained in Ref. [22]. In that paper the heat kernel coefficients have been calculated for conformal case ($\xi = 1/8$ in three dimensions). In order to compare both results we have to set $\xi = 1/8$ ($\Delta = 0$) in Eq.(26) and use the formulas of Appendix A from Ref. [22] for the three dimensional case $d = 2$.

For renormalization which we shall discuss later we shall extract from zeta function (10) the asymptotic expansion (24). Because all divergences at $s \rightarrow 0$ are contained in (24), we set $s = 0$ in the remained part. After long calculation we arrive at the following formula for zeta function

$$\begin{aligned} \zeta_A^R(s - \frac{1}{2}) = & -\frac{m}{16\pi^2\beta} \left\{ B^R(\beta) \ln \beta^2 + \Omega^R(\beta) \right\} + \frac{m^{-2s}}{(4\pi)^{3/2}} \left\{ B_0^R m^4 \frac{\Gamma(s-2)}{\Gamma(s-\frac{1}{2})} \right. \\ & \left. + B_{\frac{1}{2}}^R m^3 \frac{\Gamma(s-\frac{3}{2})}{\Gamma(s-\frac{1}{2})} + B_1^R m^2 \frac{\Gamma(s-1)}{\Gamma(s-\frac{1}{2})} + B_{\frac{3}{2}}^R m + B_2^R \frac{\Gamma(s)}{\Gamma(s-\frac{1}{2})} \right\}, \end{aligned} \quad (34)$$

where

$$B^R(\beta) = \frac{1}{2} R m^4 B_0^R - R m^2 B_1^R + R B_2^R = \frac{1}{2} \beta^4 b_0^R - \beta^2 b_1^R + b_2^R. \quad (35)$$

In order to exhibit the dependence on mass the m and on the radius of sphere R , we have introduced in above formula, the dimensionless heat kernel coefficients by relations

$$b_0^R = B_0^R/R^3 = \frac{4\pi}{3\alpha}, \quad b_1^R = B_1^R/R = \frac{7}{3}\pi\alpha - \frac{4\pi}{\alpha}(\Delta - \frac{1}{12}), \quad (36)$$

$$b_2^R = B_2^R R = \pi\alpha(\Delta - \frac{1}{12} + \frac{229}{2520}\alpha^2) - \frac{2\pi}{\alpha}(\Delta^2 - \frac{1}{6}\Delta + \frac{7}{240}). \quad (37)$$

The function $\Omega^R(\beta)$ tends to a constant for $\beta \rightarrow 0$ and $\Omega^R(\beta) = -B^R(\beta) \ln \beta^2 + \sqrt{\pi} b_{5/2}^R/\beta + O(1/\beta^2)$ for $\beta \rightarrow \infty$. The details of calculation and close form of $\Omega^R(\beta)$ are outlined in Appendix A.

At this point we would like to make a comment. The origin of the term $B^R \ln \beta^2$ is the following: In the limit $m \rightarrow \infty$ the singular part of zeta function has the structure given by Eq.(25). For small value of m it has the same poles structure multiplied by β^{2s} . This is because all functions $\mathcal{Z}(p, s)$ are proportional to this degree of β as it may be seen from Eq.(22). The difference between them in the limit $s \rightarrow 0$ is $s \ln \beta^2$ multiplied by Eq.(25). Obviously that in this limit only B_0^R , B_1^R , B_2^R survive which give the logarithm contribution to Eq.(34).

IV. THE MODEL

Because the geometrical characteristics of global monopole space-time are divergent at the origin we consider the following model: The center of monopole is surrounded by sphere with radius r_0 whose interior region is cut out. It means that in our model there is no an internal structure for the global monopole. The present model reflects this peculiarity of topological defect.

In frameworks of this model, we have to take into account both solutions of radial equation of the Laplace operator, instead of only one given in Eq.(3) which is regular at origin. The eigenfunctions now have the following form

$$\Phi(\mathbf{r}) = \sqrt{\frac{\lambda}{\alpha r}} Y_{lm}(\theta, \varphi) \left\{ C_1 J_\mu\left(\frac{\lambda}{\alpha} r\right) + C_2 N_\mu\left(\frac{\lambda}{\alpha} r\right) \right\} , \quad (38)$$

where N_μ is the Bessel function of the second kind.

In this case we have two boundaries and one has to impose two boundary conditions. Let us again choose the Dirichlet boundary condition for the radial functions at spheres of radii R and r_0 :

$$C_1 J_\mu\left(\frac{\lambda}{\alpha} R\right) + C_2 N_\mu\left(\frac{\lambda}{\alpha} R\right) = 0 , \quad (39)$$

and

$$C_1 J_\mu\left(\frac{\lambda}{\alpha} r_0\right) + C_2 N_\mu\left(\frac{\lambda}{\alpha} r_0\right) = 0 . \quad (40)$$

The set of discret eigenvalues $\lambda_{l,j}$ can be found from equation below

$$J_\mu\left(\frac{\lambda_{l,j}}{\alpha}r_0\right)N_\mu\left(\frac{\lambda_{l,j}}{\alpha}R\right) - N_\mu\left(\frac{\lambda_{l,j}}{\alpha}r_0\right)J_\mu\left(\frac{\lambda_{l,j}}{\alpha}R\right) = 0 , \quad (41)$$

which is, in fact, the condition for existence of the solution (38). Therefore we obtain the following formula for zeta function instead of Eq.(6)

$$\begin{aligned} \zeta_A(s - \frac{1}{2}) = & -\frac{\cos \pi s}{\pi} \sum_{l=0}^{\infty} (2l+1) \int_m^{\infty} dk (k^2 - m^2)^{1/2-s} \\ & \times \frac{\partial}{\partial k} \ln \left(I_\mu\left(\frac{kR}{\alpha}\right) K_\mu\left(\frac{kr_0}{\alpha}\right) - K_\mu\left(\frac{kR}{\alpha}\right) I_\mu\left(\frac{kr_0}{\alpha}\right) \right) . \end{aligned} \quad (42)$$

This general expression may be essentially simplified in the limit $R/r_0 \rightarrow \infty$ which we are interested in. Taking into account that in this limit the ratio $K_\mu(kR/\alpha)/I_\mu(kR/\alpha) < \pi \exp(-2mR/\alpha)$ is exponentially small, so we may divide the expression for zeta function (42) in two parts

$$\zeta_A(s - \frac{1}{2}) = \zeta_A^R(s - \frac{1}{2}) + \zeta_A^{r_0}(s - \frac{1}{2}) , \quad (43)$$

where

$$\zeta_A^R(s - \frac{1}{2}) = -\frac{\cos \pi s}{\pi} \sum_{l=0}^{\infty} (2l+1) \int_m^{\infty} dk (k^2 - m^2)^{1/2-s} \frac{\partial}{\partial k} \ln \left(k^{-\mu} I_\mu\left(\frac{k}{\alpha}R\right) \right) , \quad (44)$$

and

$$\zeta_A^{r_0}(s - \frac{1}{2}) = -\frac{\cos \pi s}{\pi} \sum_{l=0}^{\infty} (2l+1) \int_m^{\infty} dk (k^2 - m^2)^{1/2-s} \frac{\partial}{\partial k} \ln \left(k^\mu K_\mu\left(\frac{k}{\alpha}r_0\right) \right) . \quad (45)$$

The first part is the zeta function for pointlike global monopole which we have already calculated in last section. It depends only on the boundary condition on the sphere of radius R . The second part depends on boundary condition on the inner sphere of radius r_0 . This kind of division of zeta function has been taken place for the case of thick cosmic string in Ref. [13]. It is also in qualitative agreement with [20]. Indeed, according with [20], the internal solution gives Bessel function I_μ and the external solution gives function K_μ in expression for zeta function. The first part of zeta function (44) depends on the solutions

which are internal with respect of sphere of radius R and the second part of zeta function (45) depends on the solutions which are external for sphere of radius r_0 .

Let us consider now the second expression (45). To calculate $\zeta_A^{r_0}$ we use the same approach which we have used in last section. We have to take into account the uniform expansion for modified Bessel function of second kind $K_\mu(\mu x)$ which has the form below

$$K_\mu(\mu z) = \sqrt{\frac{\pi t}{2\mu}} e^{-\mu\eta(z)} \left\{ 1 + \sum_{k=1}^{\infty} (-1)^k \frac{u_k(t)}{\mu^k} \right\} . \quad (46)$$

Differently from the uniform expansion of Bessel function of first kind given by Eq.(7), the odd degrees of μ in above formula have the opposite sign. This fact leads to the change of sign of the heat kernel coefficients with integer index, also with respect of the heat kernel coefficients which were considered in last section. Using this uniform expansion we arrive at the following formulas for the zeta function $\zeta_A^{r_0}$

$$\zeta_A^{r_0}(s - \frac{1}{2}) = N^{r_0}(s) + \frac{m^{-2s}}{(4\pi)^{3/2}\Gamma(s - \frac{1}{2})} \sum_{k=-1}^3 (-1)^k A_k(s, r_0) , \quad (47)$$

where

$$N^{r_0}(s) = -\frac{\cos \pi s}{\pi R} \sum_{l=0}^{\infty} (2l+1) \mu \alpha \int_{\beta/\mu\alpha}^{\infty} dx \left\{ x^2 - \left(\frac{\beta}{\mu\alpha} \right)^2 \right\}^{1/2-s} \quad (48)$$

$$\times \frac{\partial}{\partial x} \left\{ \ln(K_\mu(\mu x)) + \mu\eta(x) + \frac{1}{4} \ln(1+x^2) + \frac{1}{\mu} D_1(t) - \frac{1}{\mu^2} D_2(t) + \frac{1}{\mu^3} D_3(t) \right\} ,$$

and the functions $A_k(s, r_0)$ are the same as in Eqs.(12) - (16) but they depend now on the radius r_0 . Proceeding in the same way as it was done in last section, we obtain the following expression for second part of zeta function $\zeta_A^{r_0}$:

$$\zeta_A^{r_0}(s - \frac{1}{2}) = -\frac{m}{16\pi^2\beta_0} \left\{ B^{r_0}(\beta_0) \ln \beta_0^2 + \Omega^{r_0}(\beta_0) \right\} + \frac{m^{-2s}}{(4\pi)^{3/2}} \left\{ B_0^{r_0} m^4 \frac{\Gamma(s-2)}{\Gamma(s-\frac{1}{2})} \right. \quad (49)$$

$$\left. + B_{1/2}^{r_0} m^3 \frac{\Gamma(s-\frac{3}{2})}{\Gamma(s-\frac{1}{2})} + B_1^{r_0} m^2 \frac{\Gamma(s-1)}{\Gamma(s-\frac{1}{2})} + B_{3/2}^{r_0} m + B_2^{r_0} \frac{\Gamma(s)}{\Gamma(s-\frac{1}{2})} \right\} ,$$

where $\beta_0 = mr_0$ and

$$B^{r_0}(\beta_0) = \frac{1}{2} r_0 m^4 B_0^{r_0} - r_0 m^2 B_1^{r_0} + r_0 B_2^{r_0} = \frac{1}{2} \beta_0^4 b_0^{r_0} - \beta_0^2 b_1^{r_0} + b_2^{r_0} . \quad (50)$$

Contrary to the last section the heat kernel coefficients with integer number have changed the sign and they are

$$b_0^{r_0} = B_0^{r_0}/r_0^3 = -\frac{4\pi}{3\alpha}, \quad b_1^{r_0} = B_1^{r_0}/r_0 = -\frac{7}{3}\pi\alpha + \frac{4\pi}{\alpha}(\Delta - \frac{1}{12}), \quad (51)$$

$$b_2^{r_0} = B_2^{r_0}r_0 = -\pi\alpha(\Delta - \frac{1}{12} + \frac{229}{2520}\alpha^2) + \frac{2\pi}{\alpha}(\Delta^2 - \frac{1}{6}\Delta + \frac{7}{240}). \quad (52)$$

The summary heat kernel coefficients, according to Eq.(43), are the sum of B_n^R and $B_n^{r_0}$ and they are in agreement with general formulas. We have to take into account that normal vectors for sphere of radius R and r_0 have opposite directions and that the boundaries consist now of two spheres. It is easy to understand the division of zeta function in two parts given by Eq.(43), and opposite sign of the heat kernel coefficients B^R and B^{r_0} with integer indexes, by calculating B_0 and $B_{\frac{1}{2}}$. For the space between two spheres we have

$$B_0 = \int_V dV = \frac{4\pi}{\alpha} \int_{r_0}^R r^2 dr = \frac{4\pi}{3\alpha} R^3 - \frac{4\pi}{3\alpha} r_0^3 = B_0^R + B_0^{r_0}, \quad (53)$$

$$B_{\frac{1}{2}} = -\frac{\sqrt{\pi}}{2} \int_R dS - \frac{\sqrt{\pi}}{2} \int_{r_0} dS = -2\pi^{3/2} R^2 - 2\pi^{3/2} r_0^2 = B_{\frac{1}{2}}^R + B_{\frac{1}{2}}^{r_0}. \quad (54)$$

Therefore the full zeta function in this case has the following form

$$\begin{aligned} \zeta_A(s - \frac{1}{2}) = & -\frac{m}{16\pi^2\beta_0} \left\{ B^{r_0}(\beta_0) \ln \beta_0^2 + \Omega^{r_0}(\beta_0) \right\} - \frac{m}{16\pi^2\beta} \left\{ B^R(\beta) \ln \beta^2 + \Omega^R(\beta) \right\} \\ & + \frac{m^{-2s}}{(4\pi)^{3/2}} \left\{ B_0 m^4 \frac{\Gamma(s-2)}{\Gamma(s-\frac{1}{2})} + B_{\frac{1}{2}} m^3 \frac{\Gamma(s-\frac{3}{2})}{\Gamma(s-\frac{1}{2})} + B_1 m^2 \frac{\Gamma(s-1)}{\Gamma(s-\frac{1}{2})} + B_{\frac{3}{2}} m \right. \\ & \left. + B_2 \frac{\Gamma(s)}{\Gamma(s-\frac{1}{2})} \right\}. \end{aligned} \quad (55)$$

The close expression for Ω^{r_0} is given in Appendix A.

V. THE GROUND STATE ENERGY

In the framework of zeta function approach the ground state energy is proportional to the zeta function of Laplace operator and is given by Eq.(1). In order to analyze this energy, let us first of all, consider the ground state energy for point-like global monopole. The full energy of the system consists of two parts, namely classical part due to the boundary

and monopole background, and quantum one loop correction. The general expression for boundary contributions has been considered in Refs [16,20] and it has the following form:

$$E_{cl}^R = p_R V_R + \sigma_R S_R + F_R R + \Lambda_R + \frac{h_R}{R} . \quad (56)$$

Here $V_R = 4\pi R^3/3\alpha$ and $S_R = 4\pi R^2$ are the volume and area of spherical surface, respectively. The two parameters p_R and σ_R have simple physical means as pressure and tension of surface. The constant contribution described by parameter Λ_R may be explained by the cosmological constant [30]. The other two parameters F_R , h_R have not got a special names.

The energy of monopole background can be obtained by integrating the (t, t) component of energy momentum tensor [7]

$$E_{cl}^{gm} = - \int_0^R \frac{\eta^2 \alpha^2}{r^2} dV = -4\pi \eta^2 \alpha R . \quad (57)$$

The quantum correction, using Eq.(34), is

$$E_q^R = \frac{1}{2} M^{2s} \zeta_A^R(s - \frac{1}{2})_{s \rightarrow 0} = -\frac{m}{32\pi^2 \beta} \left\{ B^R(\beta) \ln \beta^2 + \Omega^R(\beta) \right\} + \left(\frac{M}{m} \right)^{2s} \frac{1}{16\pi^{3/2}} \\ \times \left\{ B_0^R m^4 \frac{\Gamma(s-2)}{\Gamma(s-\frac{1}{2})} - \frac{2}{3} B_{\frac{1}{2}}^R m^3 + B_1^R m^2 \frac{\Gamma(s-1)}{\Gamma(s-\frac{1}{2})} + B_{\frac{3}{2}}^R m + B_2^R \frac{\Gamma(s)}{\Gamma(s-\frac{1}{2})} \right\}_{s \rightarrow 0} , \quad (58)$$

where the heat kernel coefficients B_k^R and B^R are given by Eq.(26) and (35), respectively.

In order to obtain a well defined result for the full energy, we have to renormalize the parameters of classical part (56) according to the rules below:

$$p_R \rightarrow p_R - \left(\frac{M}{m} \right)^{2s} \frac{3m^4 b_0^R}{64\pi^{5/2}} \frac{\Gamma(s-2)}{\Gamma(s-\frac{1}{2})} , \quad \sigma_R \rightarrow \sigma_R + \frac{m^3 b_{1/2}^R}{96\pi^{5/2}} , \\ F_R \rightarrow F_R - \left(\frac{M}{m} \right)^{2s} \frac{m^2 b_1^R}{16\pi^{3/2}} \frac{\Gamma(s-1)}{\Gamma(s-\frac{1}{2})} , \quad \Lambda_R \rightarrow \Lambda_R - \frac{m b_{3/2}^R}{16\pi^{3/2}} , \\ h_R \rightarrow h_R - \left(\frac{M}{m} \right)^{2s} \frac{b_2^R}{16\pi^{3/2}} \frac{\Gamma(s)}{\Gamma(s-\frac{1}{2})} . \quad (59)$$

After this procedure we obtain the following expression for ground state energy

$$E_q^R = -\frac{m}{32\pi^2 \beta} \left\{ B^R(\beta) \ln \beta^2 + \Omega^R(\beta) \right\} . \quad (60)$$

The similar general structure of ground state energy in massless case has been obtained by Blau, Visser and Wipf [16] using the dimensional considerations only. For massive case we

find in manifest form the same structure. If we have used another scale for the mass like $M \rightarrow M/\chi$, in renormalization rules (59), the above logarithmic term $\ln \beta^2$ will be replaced by $\ln(\chi\beta)^2$.

The expression (60) is, in fact, the Casimir energy for internal part of the spherical bag in global monopole background. For small radius of the bag, this energy tends to infinity as $\ln R/R$:

$$E_q^R \sim -\frac{m}{16\pi^2\beta} b_2^R \ln \beta, \quad (61)$$

and for great radius of the bag $R \rightarrow \infty$ it tends to zero

$$E_q^R \sim -\frac{mb_{5/2}^R}{16\pi^{3/2}\beta^2}. \quad (62)$$

Using these two limits we may analyze qualitatively the dependence of the Casimir energy of the internal part of the bag on its radius. The behavior of energy is defined by two heat kernel coefficients b_2^R and $b_{5/2}^R$. Both of these coefficients are the functions of non-minimal coupling parameter ξ and metric coefficient α . In general, three kinds of different behaviors exist, which are plotted in Fig.1. It is possible to analyze the energy in general case, however we shall discuss only three cases for $\xi = 1/6, 1/8, 0$.

1. $\xi = \frac{1}{6}$. In this case the behavior may be only of I and II kinds namely, the first kind for $\alpha < 1.24$ and the second one for $\alpha > 1.24$. The coefficient b_2^R does not change its sign, but $b_{5/2}^R$ does for $\alpha = 1.24$.

2. $\xi = \frac{1}{8}$. The behavior of energy may be I, II and III kinds, the first kind is for $\alpha < 1.045$, the second kind is in the region $1.045 < \alpha < 1.17$ and the third one for $\alpha > 1.17$. In the point $\alpha = 1.045$ the coefficient $b_{5/2}^R$ change the sign, but b_2^R does not up to $\alpha = 1.17$ where it changes the sign, too.

3. $\xi = 0$. This case is similar to previously one: it is of the first kind for $\alpha < 1.016$, of the second for region $1.016 < \alpha < 1.054$, and of the third for $\alpha > 1.054$.

For $\alpha \leq 1$ and $\xi = \frac{1}{6}, \frac{1}{8}, 0$ it is possible only the first kind of behavior. In the case when $\alpha = 1$ the energy was calculated numerically in Ref. [20] and our results are in agreement

with it. In this case $b_2^R = -\pi^2 16/315$ and $b_{5/2}^R = -\pi^{3/2}/120$ the dependence may be the of first kind, only.

In the limit $R \rightarrow \infty$, the quantum correction tends to zero and the full energy contains only classical part which are due to boundary and background itself.

Let us now proceed to our model. We surround the monopole origin by spheres of radii r_0 and $R > r_0$ and consider the bosonic matter field in the space between them. We do not take into account the interior of sphere of radius r_0 , there is nothing inside it. We impose the Dirichlet boundary condition on this sphere which means that there is no flux into this region. The full energy in this case consists of five parts

$$E = E_{cl}^R + E_{cl}^{r_0} + E_{cl}^{gm} + E_q^R + E_q^{r_0} , \quad (63)$$

where

$$E_{cl}^R = p_R V_R + \sigma_R S_R + F_R R + \Lambda_R + \frac{h_R}{R} , \quad (64)$$

$$E_{cl}^{r_0} = p_{r_0} V_{r_0} + \sigma_{r_0} S_{r_0} + F_{r_0} r_0 + \Lambda_{r_0} + \frac{h_{r_0}}{r_0} , \quad (65)$$

$$E_{cl}^{gm} = -4\pi\eta^2\alpha(R - r_0) , \quad (66)$$

are the classical part of energy due to the boundaries and global monopole itself, respectively, and

$$E_q^R = \frac{1}{2} M^{2s} \zeta_A^R(s - \frac{1}{2})_{s \rightarrow 0} = -\frac{m}{32\pi^2\beta} \left\{ B^R(\beta) \ln \beta^2 + \Omega^R(\beta) \right\} + \left(\frac{M}{m} \right)^{2s} \frac{1}{16\pi^{3/2}} \\ \times \left\{ B_0^R m^4 \frac{\Gamma(s-2)}{\Gamma(s-\frac{1}{2})} - \frac{2}{3} B_{\frac{1}{2}}^R m^3 + B_1^R m^2 \frac{\Gamma(s-1)}{\Gamma(s-\frac{1}{2})} + B_{\frac{3}{2}}^R m + B_2^R \frac{\Gamma(s)}{\Gamma(s-\frac{1}{2})} \right\}_{s \rightarrow 0} , \quad (67)$$

$$E_q^{r_0} = \frac{1}{2} M^{2s} \zeta_A^{r_0}(s - \frac{1}{2})_{s \rightarrow 0} = -\frac{m}{32\pi^2\beta_0} \left\{ B^{r_0}(\beta_0) \ln \beta_0^2 + \Omega^{r_0}(\beta_0) \right\} + \left(\frac{M}{m} \right)^{2s} \frac{1}{16\pi^{3/2}} \\ \times \left\{ B_0^{r_0} m^4 \frac{\Gamma(s-2)}{\Gamma(s-\frac{1}{2})} - \frac{2}{3} B_{\frac{1}{2}}^{r_0} m^3 + B_1^{r_0} m^2 \frac{\Gamma(s-1)}{\Gamma(s-\frac{1}{2})} + B_{\frac{3}{2}}^{r_0} m + B_2^{r_0} \frac{\Gamma(s)}{\Gamma(s-\frac{1}{2})} \right\}_{s \rightarrow 0} , \quad (68)$$

are the quantum corrections. Adopting the same renormalization prescription given in Eq.(59) for parameters in E_{cl}^R and $E_{cl}^{r_0}$, one arrives at the following expression for renormalized quantum corrections

$$E_q^R = -\frac{m}{32\pi^2\beta} \left\{ B^R(\beta) \ln \beta^2 + \Omega^R(\beta) \right\} \quad (69)$$

and

$$E_q^{r_0} = -\frac{m}{32\pi^2\beta_0} \left\{ B^{r_0}(\beta_0) \ln \beta_0^2 + \Omega^{r_0}(\beta_0) \right\} . \quad (70)$$

The sum of these terms gives the Casimir energy of the field in space between the two spheres in the global monopole background. The first part we have already discussed. We may consider the second part in the same way. For small radius of sphere, $r_0 \rightarrow 0$, it tends to infinity

$$E_q^{r_0} \sim -\frac{m}{16\pi^2\beta_0} b_2^{r_0} \ln \beta_0 , \quad (71)$$

and for great radius of sphere $r_0 \rightarrow \infty$ it tends to zero

$$E_q^{r_0} \sim -\frac{mb_{5/2}^{r_0}}{16\pi^{3/2}\beta_0^2} . \quad (72)$$

Because $b_2^{r_0} = -b_2^R$ and $b_{5/2}^{r_0} = b_{5/2}^R$, the energy $E_q^{r_0}$ has different behavior at small radius r_0 . The sum of b_n^R and $b_n^{r_0}$ constitutes the whole heat kernel coefficients for this space. For this reason the three kinds of dependences of $E_q^{r_0}$ on the radius are possible, only, which are displayed in Fig.1. The same results are available for $E_q^{r_0}$ as it was obtained above for E_q^R for $\xi = 1/6, 1/8, 0$; we have to change only the left plot to right one in Fig.1. The case $b_2 = 0$ has to be analyzed numerically, however this discussion is out the scope of present paper.

For any non-zero radius of the inner cavity r_0 we have finite result. In this case the Casimir energy may be positive or negative, depends on the parameters of the theory. The main problem now is with the limit $r_0 \rightarrow 0$, which has to reproduce the topological defect itself. The energy $E_q^{r_0}$ presents divergence in this limit as $\ln r_0/r_0$. This is in contradiction with above consideration of point-like monopole. If we set the radius $r_0 = 0$ at the beginning as it was done in Sec.III we obtain zero ground state energy for $R \rightarrow \infty$. From the point of view of heat kernel coefficients we have already thrown away divergent part of B_2 using

partie finite of integral (33). In frameworks of our model this thrown part appears here as divergent at origin and the additional renormalization is needed.

For renormalization we may use the last term h_{r_0}/r_0 in the classical part of energy which is due to boundary (65). We define a parameter M_0 with dimension of mass by the relation $h_{r_0} = GM_0^2$, where G is gravitational constant. With this definition, the divergent contribution for small radius r_0 may be canceled by the renormalization rule below

$$M_0^2 \rightarrow M_0^2 + \frac{m_{pl}^2}{32\pi^2} \{2B^{r_0} \ln(mr_0) + \Omega^{r_0}(mr_0)\} , \quad (73)$$

where m_{pl}^2 is the Plank mass, and then ground state energy is zero.

VI. CONCLUSION

In this paper we have considered the ground state energy of quantum scalar field in the background of global monopole space-time, with line element given by Eq.(2), in framework of zeta function approach. In order to reveal the role of singularity, we investigated two cases: In the first case we calculated ground state energy for point-like global monopole. We surrounded the origin by sphere of radius R and obtained that the ground state energy of the field inside. It has the form (60) and tend to zero in the limit $R \rightarrow \infty$ and to infinity when $R \rightarrow 0$. The behavior of this Casimir energy in this cases is managed by two heat kernel coefficients b_2^R and $b_{5/2}^R$, respectively. The qualitative plots of the ground state energy for different values of the parameters $\xi = 1/6, 1/8, 0$ and α are given in Fig.1(a). For $\alpha \leq 1$ it may be only in first kind for above values of ξ .

In order to avoid the problem with singularity at origin, we investigated the second case in Sec.IV, the following model: We surrounded the origin by a sphere of radius $r_0 < R$ and considered the scalar field in domain between these two spheres using the Dirichlet boundary condition on the wave function associated with the massive scalar field, on the two surfaces. This boundary condition guarantee that there is no flux of particle through the spherical surfaces. The Casimir energy in this case consists of two parts given in Eqs.(69), (70). The

first part is the same as for pint-like monopole case and second one is due to the inner sphere of radius r_0 around origin. The structure of second part of ground state energy (70) is similar: there is logarithmic divergence at origin which tends to zero for infinite radius. The sign of energy for small distance is opposite. For this reason there appears three kind of dependence of energy which are displayed in Fig.1(b).

In the limit $R \rightarrow \infty$ and finite $r_0 \neq 0$ only one contribution in ground state energy survives (69). In the limit $r_0 \rightarrow 0$, it is divergent and additional renormalization is needed which is given by Eq.(73). After this renormalization the ground state energy of a global monopole will be zero. This is in agreement with ground state energy of point-like global monopole.

If one fills up this cavity around the origin by matter, the situation becomes different. We may expect the same kind of divergence for additional energy from the interior of monopole but with opposite sign. We have already seen that the internal and external contributions have opposite signs: $b_2^R = -b_2^{r_0}$. For this reason we may expect that this kind of divergence will cancel, however we cannot say anything analytically about divergence $\Omega(mr_0)/r_0$. In flat space-time [20] it cancels, too, because the ground state energy is zero for zero radius of the bag. The same cancellation takes place in the case of a thick cosmic string background considered in Ref. [13]. All of these aspects will be discussed in a separate paper.

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APPENDIX A:

In this appendix we want to give some brief explanation about the most important results found by us. First of all we represent the expressions for A_k as series in powers of $\beta = mr_0$ regarding for a moment $b, \beta < 1$ and $\Delta > -1/4$ for convergence of series. They are

$$A_{-1}(s, R) = \frac{m}{\beta} \beta^{2s} \frac{4\pi}{\alpha} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \beta^{2n} \frac{\Gamma(s-1+n)}{s+n-\frac{1}{2}} \sum_{l=0}^{\infty} \frac{\nu_l}{(\nu_l^2 + \Delta)^{s+n-1}}, \quad (\text{A1})$$

$$A_0(s, R) = -\frac{m}{\beta} \beta^{2s} 4\pi^{3/2} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} b^{2n} \Gamma(s+n-\frac{1}{2}) \zeta_H(2s+2n-2, \frac{1}{2}), \quad (\text{A2})$$

$$A_k(s, R) = -\frac{m}{\beta} \beta^{2s} 16\pi^{3/2} \alpha^k \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} b^{2n} \sum_{a=0}^k x_{k,a} \frac{\Gamma(s+\frac{k}{2}+a+n-\frac{1}{2})}{\Gamma(\frac{k}{2}+a)} \times \sum_{l=0}^{\infty} \frac{(\nu_l^2 + \Delta)^a}{\nu_l^{2s+k+2a+2n-2}}. \quad (\text{A3})$$

In the above formulas we have used the following notations : $\nu_l = l + \frac{1}{2}$, $\Delta = 2(1 - \alpha^2)(\xi - 1/8)$ and $b^2 = \beta^2 + \Delta$. As we can see only the first three terms in the expression for A_{-1} , with $n = 0, 1, 2$, two terms in A_0 , A_1 , A_2 and one term in A_3 are divergent in the limit $s \rightarrow 0$. Extracting these terms we may set $s = 0$ in the remaining series and we get the following result:

$$\begin{aligned} \zeta_A^R(s - \frac{1}{2}) &= N^R(s, \beta) + \frac{m^{-2s}}{(4\pi)^{3/2} \Gamma(s - \frac{1}{2})} \sum_{k=-1}^3 A_k(s, R) \\ &= \frac{m^{-2s}}{(4\pi)^{3/2} \Gamma(s - \frac{1}{2})} \beta^{2s} \left\{ m^4 B_0^R \Gamma(s-2) + m^3 B_{\frac{1}{2}}^R \Gamma(s - \frac{3}{2}) + m^2 B_1^R \Gamma(s-1) \right. \\ &\quad \left. + m B_{\frac{3}{2}}^R \Gamma(s - \frac{1}{2}) + B_2^R \Gamma(s) \right\} - \frac{1}{16\pi^2 R} \left\{ \sum_{k=-1}^3 \omega_k(\beta) + \omega_f^R(\beta) \right\}, \end{aligned} \quad (\text{A4})$$

where

$$\begin{aligned} \omega_f^R(\beta) &= 32\pi \sum_{l=0}^{\infty} \nu_l \sqrt{\nu_l^2 + \Delta} \int_{\beta/\sqrt{\nu_l^2 + \Delta}}^{\infty} dx \sqrt{x^2 - \frac{\beta^2}{\nu_l^2 + \Delta}} \\ &\quad \times \frac{\partial}{\partial x} \left\{ \ln I_{\mu}(\mu x) - \mu \eta(x) + \frac{1}{4} \ln(1+x^2) - \frac{1}{\mu} D_1 - \frac{1}{\mu^2} D_2 - \frac{1}{\mu^3} D_3 \right\}, \quad (\text{A5}) \\ \omega_{-1}(\beta) &= -\frac{4\pi}{\alpha} \left\{ \left[-\frac{7}{2} \zeta'(-3) - \frac{7}{160} + \frac{1}{240} \ln 2 + \Delta \left(-2\zeta'(-1) + \frac{1}{6} - \frac{1}{6} \ln 2 \right) \right] \right. \\ &\quad \left. + \beta^2 \left[2\zeta'(-1) + \frac{1}{4} + \frac{1}{6} \ln 2 + \Delta (2\gamma + 4 \ln 2 - 3) \right] + \beta^4 \left[\frac{1}{3} \gamma + \frac{2}{3} \ln 2 - \frac{13}{36} \right] \right\} \end{aligned}$$

$$\begin{aligned}
& - \sum_{n=3}^{\infty} \frac{(-1)^n}{n!} \Gamma(n-1) \Delta^n \zeta_H(2n-3, \frac{1}{2}) - 2\beta^2 \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \Delta^n \Gamma(n) \zeta(2n-1) \\
& + \frac{1}{3} \beta^4 Y_{1,1}(\Delta) + \sum_{n=3}^{\infty} \frac{(-1)^n}{n!} \frac{\Gamma(n-1)}{n-\frac{1}{2}} \beta^{2n} \sum_{l=0}^{\infty} \frac{\nu_l}{(\nu_l^2 + \Delta)^{n-1}} \Big\} , \tag{A6}
\end{aligned}$$

$$\omega_0(\beta) = -2\pi^2 \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \frac{\Gamma(n-\frac{1}{2})}{\Gamma(\frac{3}{2})} b^{2n} \zeta_H(2n-2, \frac{1}{2}) , \tag{A7}$$

$$\begin{aligned}
\omega_1(\beta) = & -2\pi\alpha \left\{ \left[-\zeta'(-1) - \frac{5}{36} - \frac{1}{12} \ln 2 + \Delta \left(-\gamma - 2 \ln 2 + \frac{5}{3} \right) \right] \right. \\
& + \beta^2 \left[\frac{7}{3} \gamma + \frac{1}{2} + \frac{14}{3} \ln 2 \right] + \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \left(\Gamma(n) - \frac{10}{3} \Gamma(n+1) \right) b^{2n} \zeta_H(2n-1, \frac{1}{2}) \\
& \left. - \frac{10}{3} \Delta Y_{1,1}(b) \right\} , \tag{A8}
\end{aligned}$$

$$\begin{aligned}
\omega_2(\beta) = & -2\pi^2 \alpha^2 \left\{ \frac{3}{16} \pi^2 \Delta + \frac{5}{32} \pi^2 \Delta^2 + \frac{1}{2} Y_{\frac{1}{2},0}(b) - \frac{3}{2} Y_{\frac{3}{2},0}(b) + \frac{15}{16} Y_{\frac{5}{2},0}(b) \right. \\
& \left. + \Delta \left[-\frac{3}{2} Y_{\frac{3}{2},2}(b) + \frac{15}{8} Y_{\frac{5}{2},2}(b) \right] + \Delta^2 \frac{15}{16} Y_{\frac{5}{2},4}(b) \right\} , \tag{A9}
\end{aligned}$$

$$\begin{aligned}
\omega_3(\beta) = & -2\pi\alpha^3 \left\{ \frac{293}{1512} - \frac{22}{2520} \gamma - \frac{229}{1260} \ln 2 + \left[\frac{25}{24} X_{1,1}(b) - \frac{177}{20} X_{2,1}(b) \right. \right. \\
& + \frac{221}{15} X_{3,1}(b) - \frac{442}{63} X_{4,1}(b) \Big] + \Delta \left[-\frac{177}{20} Y_{2,3}(b) + \frac{442}{15} Y_{3,3}(b) - \frac{442}{21} Y_{4,3}(b) \right] \\
& \left. + \Delta^2 \left[\frac{221}{15} Y_{3,5}(b) - \frac{442}{21} Y_{4,5}(b) \right] - \Delta^3 \frac{442}{63} Y_{4,7}(b) \right\} , \tag{A10}
\end{aligned}$$

$$X_{p,q}(b) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{\Gamma(n+p)}{\Gamma(p)} b^{2n} \zeta_H(2n+q, \frac{1}{2}) , \tag{A11}$$

$$Y_{p,q}(b) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{\Gamma(n+p)}{\Gamma(p)} b^{2n} \zeta_H(2n+q, \frac{1}{2}) . \tag{A12}$$

Each of the above series may be analytically continued in terms of digamma function Ψ for arbitrary values of b and Δ . For example

$$\sum_{n=3}^{\infty} \frac{(-1)^n}{n!} \frac{\Gamma(n-1)}{n-\frac{1}{2}} \beta^{2n} \sum_{l=0}^{\infty} \frac{\nu_l}{(\nu_l^2 + \Delta)^{n-1}} = -2\beta^4 \int_0^1 dx x(1-x)^2 \tag{A13}$$

$$\begin{aligned}
& \times \left\{ \Psi \left[\frac{1}{2} - i\sqrt{\Delta + \beta^2 x^2} \right] + \Psi \left[\frac{1}{2} + i\sqrt{\Delta + \beta^2 x^2} \right] - \Psi \left[\frac{1}{2} - i\sqrt{\Delta} \right] - \Psi \left[\frac{1}{2} + i\sqrt{\Delta} \right] \right\} , \\
& \sum_{n=3}^{\infty} \frac{(-1)^n}{n!} \Gamma(n-1) \Delta^n \zeta_H(2n-3, \frac{1}{2}) \\
& = -\Delta^2 \int_0^1 dx x(1-x^2) \left\{ \Psi \left[\frac{1}{2} - i\sqrt{\Delta} \right] + \Psi \left[\frac{1}{2} + i\sqrt{\Delta} \right] - 2\Psi \left[\frac{1}{2} \right] \right\} . \tag{A14}
\end{aligned}$$

This kind of representation is suitable for numerical calculations.

Adding and subtracting the asymptotic expansion of zeta function (25) we obtain the following formula

$$\zeta_A^R(s - \frac{1}{2}) = -\frac{m}{16\pi^2\beta} \left\{ B^R(\beta) \ln \beta^2 + \Omega^B(\beta) \right\} + \frac{m^{-2s}}{(4\pi)^{3/2}} \left\{ B_0^R m^4 \frac{\Gamma(s-2)}{\Gamma(s-\frac{1}{2})} \right. \\ \left. + B_{\frac{1}{2}}^R m^3 \frac{\Gamma(s-\frac{3}{2})}{\Gamma(s-\frac{1}{2})} + B_1^R m^2 \frac{\Gamma(s-1)}{\Gamma(s-\frac{1}{2})} + B_{\frac{3}{2}}^R m + B_2^R \frac{\Gamma(s)}{\Gamma(s-\frac{1}{2})} \right\}, \quad (\text{A15})$$

where $\Omega^B(\beta) = \sum_{k=-1}^3 \omega_k(\beta) + \omega_f^R(\beta)$. It is easy to see that the function $\Omega^B(\beta)$ tends to a constant which may be calculated using the above formulas. Indeed, in the limit $\beta \rightarrow \infty$ we have to obtain asymptotic expansion of zeta function. Because one has already extracted the first five terms, so the next term will be $B_{5/2}$. For this reason we get the following behavior for great β : $\Omega^B(\beta) \sim -B^R \ln \beta^2 + \sqrt{\pi} b_{\frac{5}{2}}^R / \beta + \dots$. The coefficient $b_{\frac{5}{2}}^R$ may be found in the same way and it has the following form

$$b_{\frac{5}{2}}^R = \frac{R}{m} B_{\frac{5}{2}}^R = \pi^{3/2} \left\{ \frac{1}{16} \alpha^4 + \frac{1}{2} \left(\Delta - \frac{1}{12} \right) \alpha^2 - \left(\Delta^2 - \frac{1}{6} \Delta + \frac{7}{240} \right) \right\}. \quad (\text{A16})$$

It is easy to obtain the formulas for zeta function $\zeta_A^{r_0}$ from above expressions. The index k in A_k corresponds to a term which is proportional to μ^{-k} in uniform expansion of the Bessel function in Eq.(8). The uniform expansion of the modified Bessel function of second kind given in Eq.(46) may be obtained from uniform expansion of the modified Bessel function of first kind (7) by replacing the index μ to $-\mu$. For this reason in order to obtain the formulas for the zeta function $\zeta_A^{r_0}$, we may replace $R \rightarrow r_0$, $\beta \rightarrow \beta_0$ and $A_k(s, R) \rightarrow (-1)^k A_k(s, r_0)$ in the above formulas for zeta function ζ_A^R . As the odd degrees of μ give contributions to heat kernel coefficients with integer index, they will change the sign. Therefore we have the following formula for the zeta function $\zeta_A^{r_0}$

$$\zeta_A^{r_0}(s - \frac{1}{2}) = -\frac{m}{16\pi^2\beta_0} \left\{ B^{r_0}(\beta_0) \ln \beta_0^2 + \Omega^{r_0}(\beta_0) \right\} + \frac{m^{-2s}}{(4\pi)^{3/2}} \left\{ B_0^{r_0} m^4 \frac{\Gamma(s-2)}{\Gamma(s-\frac{1}{2})} \right. \\ \left. + B_{\frac{1}{2}}^{r_0} m^3 \frac{\Gamma(s-\frac{3}{2})}{\Gamma(s-\frac{1}{2})} + B_1^{r_0} m^2 \frac{\Gamma(s-1)}{\Gamma(s-\frac{1}{2})} + B_{\frac{3}{2}}^{r_0} m + B_2^{r_0} \frac{\Gamma(s)}{\Gamma(s-\frac{1}{2})} \right\}, \quad (\text{A17})$$

where $\Omega^{r_0}(\beta_0) = \sum_{k=-1}^3 (-1)^k \omega_k(\beta_0) + \omega_f^{r_0}(\beta_0)$ and

$$\omega_f^R(\beta) = 32\pi \sum_{l=0}^{\infty} \nu_l \sqrt{\nu_l^2 + \Delta} \int_{\beta/\sqrt{\nu_l^2 + \Delta}}^{\infty} dx \sqrt{x^2 - \frac{\beta^2}{\nu_l^2 + \Delta}} \\ \times \frac{\partial}{\partial x} \left\{ \ln K_{\mu}(\mu x) + \mu \eta(x) + \frac{1}{4} \ln(1+x^2) + \frac{1}{\mu} D_1 - \frac{1}{\mu^2} D_2 + \frac{1}{\mu^3} D_3 \right\}. \quad (\text{A18})$$

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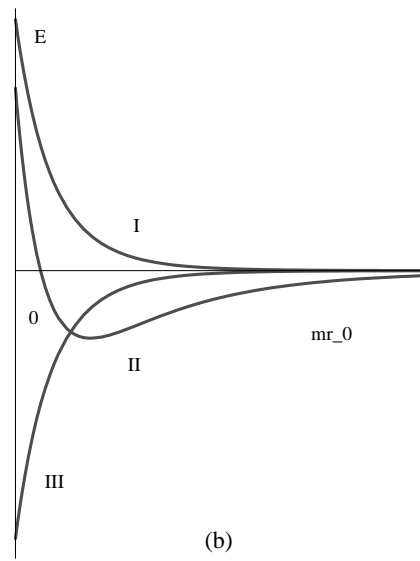
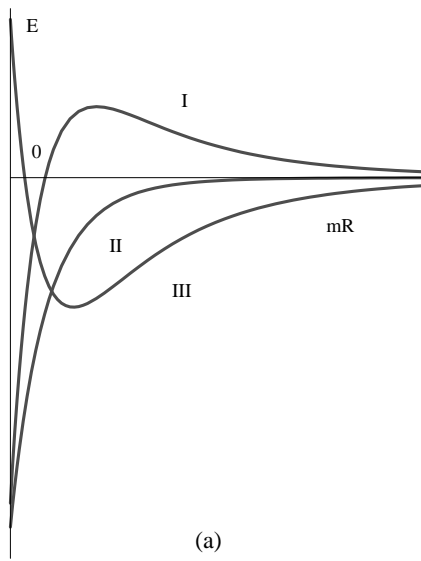
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Figure captions

Figure 1. Three types of dependence of the ground state energy for the field inside the sphere of radius R (a) and outside the sphere of radius r_0 (b).

Figures



Footnotes

* e-mail:emello@fisica.ufpb.br

† e-mail: valdir@fisica.ufpb.br

‡ On leave from Kazan State Pedagogical University, Kazan, Russia;
e-mail:nail@ntp.ksu.ras.ru